THEORETICAL STUDIES OF RARE-EARTH NUCLEI LEADING TO $_{50}$Sn-DAUGHTER PRODUCTS AND THE ASSOCIATED SHELL EFFECTS

Sushil Kumar [1]

^{a}*Department of Applied Science, Chitkara University, Solan -174103,(H.P.) India.*

Cluster decays of rare-earth nuclei are studied with a view to look for neutron magic shells for the $_{50}$Sn nucleus as the daughter product always. The ^{100}Sn and ^{132}Sn radioactivities are studied to find the most probable cluster decays and the possibility, if any, of new neutron shells. For a wide range of parent nuclei considered here (from Ba to Pt) ^{12}C from ^{112}Ba and ^{78}Ni from ^{210}Pt parent are predicted to be the most probable clusters (minimum decay half-life) referring to ^{100}Sn and ^{132}Sn daughters, respectively. Also, ^{22}Mg decay of ^{122}Sm is indicated at the second best possibilty for ^{100}Sn-daughter decay. In addition to these well known magic shells (Z=50, N=50 and 82), a new magic shell at Z=50, N=66 (^{116}Sn daughter) is indicated for the ^{64}Ni decay from ^{180}Pt parent.

[1]Email: sushilk17@gmail.com

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1 Introduction

Since the discovery of ^{14}C-decay from ^{223}Ra by Rose and Jones [1] in 1984, many other ^{14}C-decays from other radioactive nuclei (^{221}Fr, ^{221,222,224,226}Ra, ^{223,225}Ac and ^{226}Th) and some 12 to 13 neutron-rich clusters, such as ^{20}O, ^{23}F, $^{22,24-26}$Ne, ^{28,30}Mg and ^{32,34}Si, have been observed experimentally for the ground-state decays of translead ^{226}Th to ^{242}Cm parents [2–5], which all decay with the doubly closed shell daughter ^{208}Pb (Z=82, N=126) or its neighboring nuclei. Theoretically, such an exotic natural radioactivity of emitting particles (nuclei) heavier than α-particle was already predicted in 1980 by Săndulescu, Poenaru and Greiner [6] on the basis of the quantum mechanical fragmentation theory (QMFT) proposed by [7, 8]. Todate, ^{34}Si is the heaviest cluster observed with the longest decay half-life ever measured ($log_{10}T_{1/2}(s) = 29.04$) from ^{238}U parent [9]. Recently, Poenaru et.al., extended the region of possible emitted clusters $A_c = 14 - 34$ measured in the region of emitters with $Z = 87$ - 96 to superheavy elements up to 124 [10]. In this systematic heavy particle radioactivity they consider not only the emitted cluters with atomic numbers $2 < Z_c < 29$ but also heavier ones up to $Z_c = Z - 82$, around ^{208}Pb a doubly magic daughter. For this purpose they used Analytical Superasymetric Fission (ASAF) model and estimated the half- life for ^{128}Sn emission from ^{256}Fm (Q-value = 252.129 MeV) and for ^{130}Te emission from ^{262}Rf (Q-value = 274.926 MeV): $log_{10}T^{Fm}(s)$ = 4.88 and $log_{10}T^{Rf}(s)$= 0.53, respectively. They are in agreement with experimental values for spontaneous fission [11]: 4.02 and 0.32, respectively.

Keeping in mind the doubly magic nature of the ^{208}Pb daughter, a second island of heavy-cluster radioactivity was predicted on the basis of analytical superasymmetric fission model (ASAFM)[12] and preformed cluster model (PCM)[13], in the decays of some neutron-deficient rare-earth nuclei in to ^{100}Sn (Z=N=50) daughter or a neighboring nucleus. Furthermore, Kumar et al., [13] predicted another doubly closed ^{132}Sn (Z=50,N=82) daughter radioactivity, for decays of some selective neutron-rich rare-earth nuclei. More ecently, an unexpected increase in E2 strengths has been reported between the midshell isotope ^{116}Sn (Z=50, N=66) and its lighter neighbor, ^{114}Sn [14], also a new shell closure at N=90 is predicted for the ^{140}Sn isotope on the basis of shell model calculations [15]. Experimentally, several unsuccessful attempts [16–19] have been made to measure the ^{100}Sn-daughter radioactivity from the ^{114}Ba parent nucleus produced in ^{58}Ni+^{58}Ni reaction. Instead, a new phenomenon of intermediate mass fragments (IMFs,

with $3 \leq Z \leq 9$), also referred to as 'clusters' or 'complex fragments', emitted from the excited compound nucleus, was also observed [20]. It is worth mentioning that intermediate mass fragments are mostly found in reactions at intermediate incident energies where colliding nuclei breaks into many pieces[21].

In this paper,heavy cluster emissions of rare-earth parents (329 cases) with $_{50}$Sn always as the daughter product is considered. The new experimental mass table [22] and the theoretical masses [23] are used to determine the released energy. Specifically, emission of various isotopes of C, O, Ne, Mg, Si, S, Ar, Ca, Ti, Cr, Fe and Ni, are considered respectively, from neutron-deficient to neutron-rich Ba, Ce, Nd, Sm, Gd, Dy, Er, Yb, Hf, W, Os and Pt parents, with a view to look for ^{100}Sn and ^{132}Sn radioactivities, as well as any other new Sn radioactivity with new shell closures in neutrons. Since the cluster decays are more probable with daughters as magic nuclei, the decay half-lives are expected to drop (be minimum) for the magic daughters. The same idea was utilized earlier for the (spherical) sub-shell closed $_{40}$Zr daughter [24, 25], including also a brief report of the results on $_{50}$Sn daughter [24]. This calculation is based on the preformed cluster model (PCM) [26, 27], described briefly in Sect. II. The results of our calculation are presented in Sect. III and a summary of our results in Sect. IV.

2 The model

The PCM model [26] uses the dynamical collective coordinates of mass (and charge) asymmetry, $\eta = (A_1 - A_2)/(A_1 + A_2)$ and $\eta_Z = (Z_1 - Z_2)/(Z_1 + Z_2)$, first introduced in the QMFT [7,8], which are in addition to the usual coordinates of relative separation R and deformations β_{2i} $(i = 1, 2)$ of two fragments. Then, in the standard approximation of decoupled R and η motions, in PCM, the decay constant λ or the decay half-life $T_{1/2}$ is defined as

$$\lambda = \frac{ln2}{T_{1/2}} = P_0 P \nu_0, \tag{1}$$

Here P_0 is the cluster (and daughter) preformation probability and P, the barrier penetrability, which refer to the η and R motions, respectively. ν_0 is the barrier assault frequency. The P_0 are the solutions of the stationary Schrödinger equation in η,

$$\left\{ -\frac{\hbar^2}{2\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \frac{1}{\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} + V_R(\eta) \right\} \psi^{(\omega)}(\eta) = E^{(\omega)} \psi^{(\omega)}(\eta), \tag{2}$$
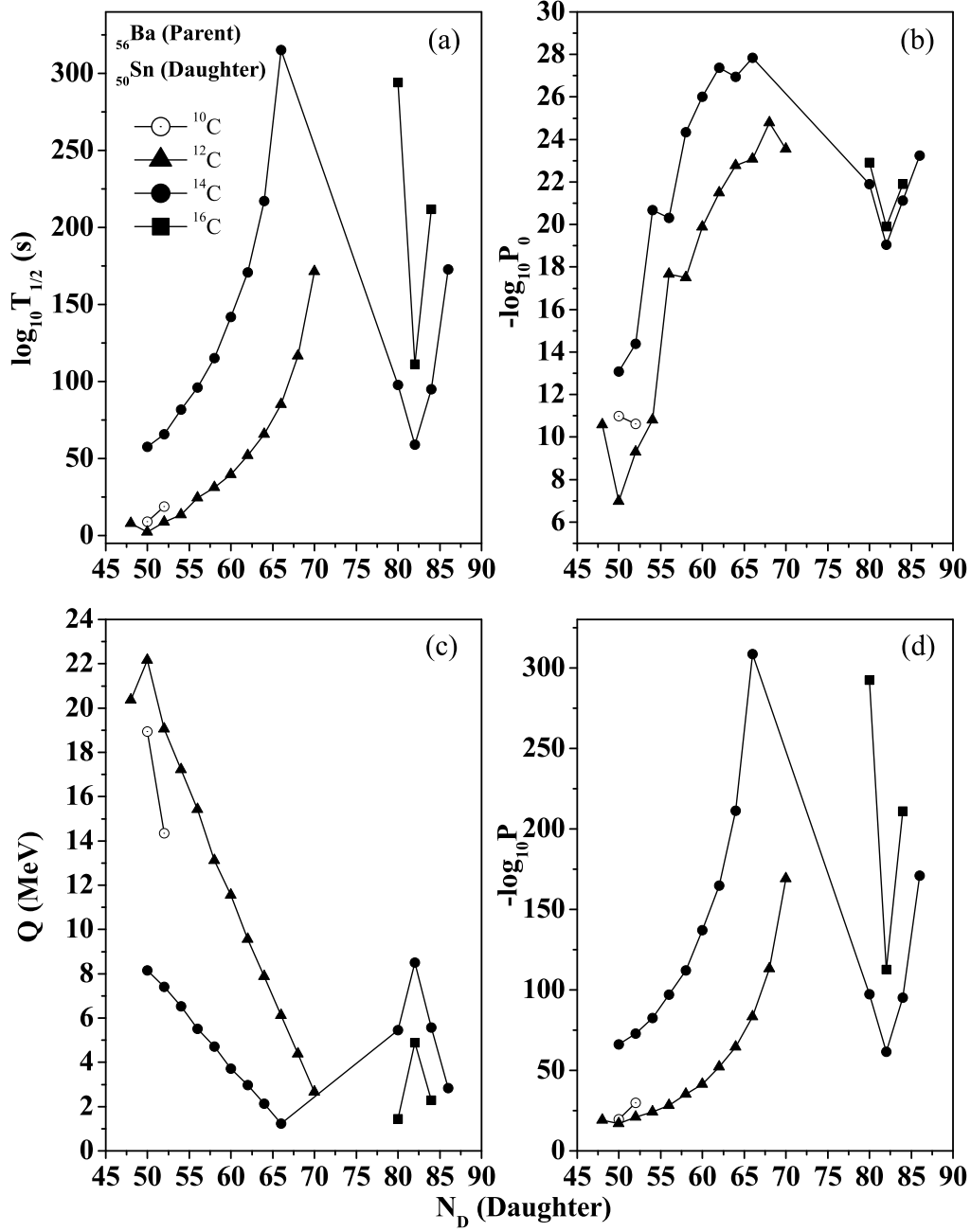
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Figure 1: The decay half-lives $T_{1/2}(s)$ and other characteristic quantities like preformation factors P_0, Q-values (in MeV), and penetrabilities P of different Carbon clusters emitted with $_{50}$Sn daughters from various isotope of Ba nuclei, calculated on the basis of the PCM, plotted as a function of daughter neutron number N_D.

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which on proper normalization gives with ω=0,1,2,3... . Eq. (2) is solved at a fixed $R = R_a = C_t(= C_1 + C_2)$ (the first turning point of the WKB integral, defined below), where C_i are the Süssmann central radii $C_i = R_i - (1/R_i)$ (in fm), with the radii $R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}$. Many other radius formulas are available [28] and widely used for the calculations of barrier heights, is also a subject of interest for the future study in PCM.

$$P_0 = \sqrt{B_{\eta\eta}} \mid \psi^{(0)}(\eta(A_i)) \mid^2 (2/A), \tag{3}$$

The fragmentation potential $V_R(\eta)$ in (2) is calculated simply as the sum of the Coulomb interaction, the nuclear proximity potential [29] and the ground state binding energies of two nuclei,

$$V(R_a, \eta) = -\sum_{i=1}^{2} B(A_i, Z_i) + \frac{Z_1 Z_2 e^2}{R_a} + V_P, \tag{4}$$

The proximity potential between two nuclei is defined as

$$V_p = 4\pi\overline{C}\gamma b\Phi(\xi) \tag{5}$$

here γ is the nuclear surface tension coefficient, $\overline{C}$ determines the distance between two points of the surfaces, evaluated at the point of closest approach and $\Phi(\xi)$ is the universal function, since it depends only on the distance between two nuclei, and is given as

$\Phi(\xi) = -0.5(\xi - 2.54)^2 - 0.0852(\xi - 2.54)^3,$

$\quad for \quad \xi \leq 1.2511$

$\quad = (-3.437exp(-\xi/0.75).for \quad \xi \geq 1.2511$

Here, ξ= s/b, i.e s in units of b, with the separation distance s=$R - C_1 - C_2$. b is the diffuseness of the nuclear surface, given by

$$b = \left[\pi/2\sqrt{3}\ln 9\right]_{t_{10-90}} \tag{6}$$

where t_{10-90} is the thickness of the surface in which the density profile changes from 90% to 10%. The γ is the specific nuclear surface tension, given by

$$\gamma = 0.9517\left[1 - 1.7826\left(\frac{N-Z}{A}\right)^2\right]MeVfm^{-2}. \tag{7}$$

In recent years, many more microscopic potentials are available that takes care various aspects such as overestimation of fusion barrier in original proximity potential, isospin

effects. A comparison is also available between all models [30]. As noted above, even modified proximity potentials were also given. We plan to study cluster decay with these new proximity potentials in near future. Here B's are taken from the 2003 experimental compilation of Audi and Wapstra [22] and, whenever not available in [22], from the 1995 calculations of Möller *etal.* [23]. Thus, full shell effects are contained in our calculations that come from the experimental and/or calculated binding energies.We also note that for exotic clusters/nuclei with neutron/proton rich matter, new binding energies are also available [31]. The momentum dependent potentials and symmtry energy potential which are found to have drastic effect at higher densities will not affect decay studies, since these happens at lower tale of the density [32, 33]. Here in Eq. (4), the Coulomb and proximity potentials are for spherical nuclei, and charges Z_1 and Z_2 in (4) are fixed by minimizing the potential in η_Z coordinate. The mass parameters $B_{\eta\eta}(\eta)$, representing the kinetic energy part in Eq. (2), are the classical hydrodynamical masses of Kröger and Scheid [34], used here for simplicity.

The penetrability P is the WKB tunnelling integral, solved analytically [26] for the second turning point R_b defined by $V(R_b)$=Q-value for the ground-state decay, and the assault frequency ν_0 in (1) is given simply as

$$\nu_0 = (2E_2/\mu)^{1/2}/R_0, \tag{8}$$

with $E_2 = (A_1/A)Q$, the kinetic energy of cluster (the lighter fragment), for the Q-value shared between the two products as inverse of their masses. R_0 is the radius of parent nucleus, and μ, the reduced mass.

3   Results and discussion

As already stated in the introduction, the cluster decays of various isotopes of $_{56}$Ba to $_{78}$Pt parents are calculated for the daughter nucleus to be always an isotope of $_{50}$Sn nucleus. For example, for the neutron-deficient $^{110-132}$Ba and neutron-rich $^{144-150}$Ba parents considered here, different isotopes of Carbon cluster would give rise to various isotopes of $_{50}$Sn daughter. This is illustrated in Fig. 1 for the decay half-life $T_{1/2}$ of various C-decays, together with the Q-values, logarithms of penetrability P and preformation factor P_0, as a function of N_D, the neutron number of $_{50}$Sn daughter. The impinging frequency ν_0 is nearly constant $\sim 10^{21}(s^{-1})$. All the four quantities Q, P, P_0, and $T_{1/2}$ show the shell
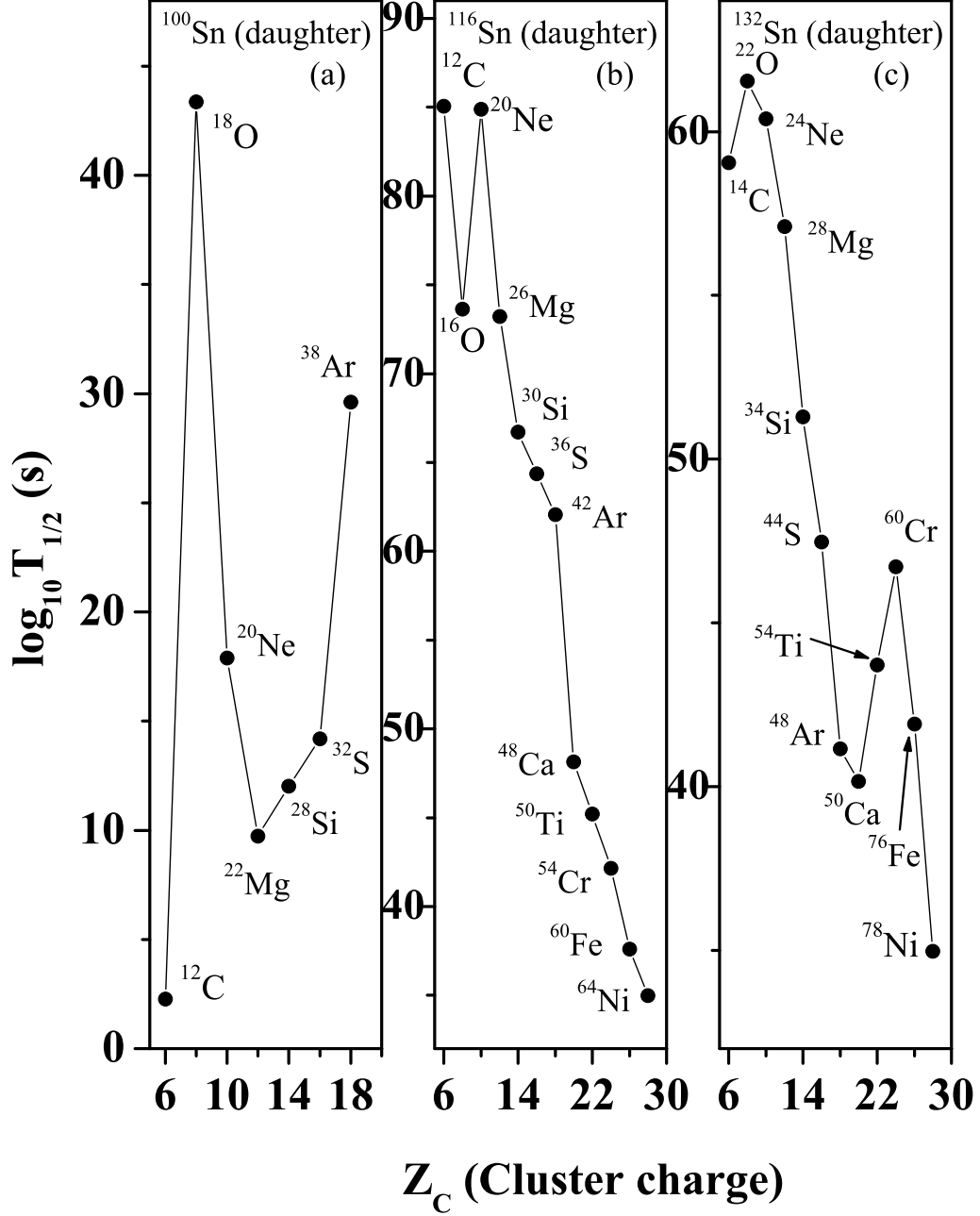
Figure 2: $log_{10}T_{1/2}(s)$ for the most probable clusters emitted from various Ba to Pt parents with (a) ^{100}Sn (b) ^{116}Sn and (c) ^{132}Sn daughter, calculated on the basis of the PCM, plotted as a function of cluster proton number Z_2. Note the different ordinate-scales used in these figures.

effects at magic $N_D=50$ and 82; the Q, P and P_0 being large and $T_{1/2}$ small at these numbers. Thus, the most favorable decay is ^{12}C from ^{112}Ba nucleus in the $48 \leq N_D \leq 70$ region, leaving behind ^{100}Sn as the daughter product, and the ^{14}C cluster from ^{146}Ba in the $72 \leq N_D \leq 86$ region with ^{132}Sn as the daughter product. This result is same as in Refs. [13] where the most probable clusters for ^{100}Sn daughters were obtained as the $A_2 = 4n$, N=Z, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg and ^{28}Si, emitted from the respective Ba to Gd parents, and that these were the ^{14}C, ^{20}O, etc., for ^{132}Sn daughter, emitted from ^{146}Ba, ^{152}Ce, etc.

In the present study, however, the other most probable clusters considered are (isotopes of O, Ne, Mg, Si, S, Ar, Ca, Ti, Cr, Fe and Ni) from heavier neutron-deficient and neutron-rich rare-earth parents ($^{118-170}$Ce, $^{118-176}$Nd, $^{122-184}$Sm, $^{132-190}$Gd, $^{132-194}$Dy, $^{138-200}$Er, $^{148-200}$Yb, $^{154-208}$Hf, $^{156-208}$W, $^{160-210}$Os and $^{168-210}$Pt). Interestingly, ^{12}C remains to be the most favorable cluster-decay from ^{112}Ba parent with ^{100}Sn-daughter [13], but for ^{132}Sn-daughter the most favorable cluster is now ^{78}Ni from ^{210}Pt, instead of ^{14}C from ^{146}Ba. This is illustrated in Fig. 2(a) and (c), respectively, for ^{100}Sn and ^{132}Sn daughters, where the most probable clusters emitted from Ba to Pt parents are plotted. The fact that the most probable cluster ^{78}Ni, arising from Pt parents, occur at $N_D=82$ of the $_{50}$Sn daughter is illustrated in Fig. 3 for $T_{1/2}$ alone. However, in Fig. 3, in addition to the strong minima at $_{50}$Sn-daughter neutrons $N_D=82$, a new minimum is also shown to be persent at $N_D=66$ for the $_{50}$Sn-daughter, emitting ^{64}Ni cluster from ^{180}Pt parent. This is further illustrated to be true in Fig. 2(b). Thus, a new possibility of ^{116}Sn-daughter radioactivity is indicated here. Apparently, other cases of interest in Fig. 2 are the ^{22}Mg decay of ^{122}Sm and ^{50}Ca decay of ^{182}Yb, respectively, with ^{100}Sn and ^{132}Sn daughters.

Finally, Fig. 4 gives a complete histogram of the decay half-lives $log_{10}T_{1/2}$(s) as a function of the neutron number N_D of the emitted $_{50}$Sn-daughters with the most probable clusters (minimum $T_{1/2}$ values) from some 329 parents taken from Ba to Pt with mass numbers $A = 110 - 210$. Limitation to $N_D \sim 94$ since, for $N_D > 90$, the contribution from nuclei heavier than Pt would also become important. Note that in Fig. 4, $_{50}$Sn-daughter is kept fixed and all possible clusters are considered from different parents (total 1617 combinations with $_{50}$Sn-daughter and the probable cluster), and then the one with minimum half-life time is plotted. Apparently, the shortest half-life time $log_{10}T_{1/2}(s) = 2.27$ (with Q-value=22.16 MeV) is obtained for ^{12}C decay of ^{112}Ba. The role of magic
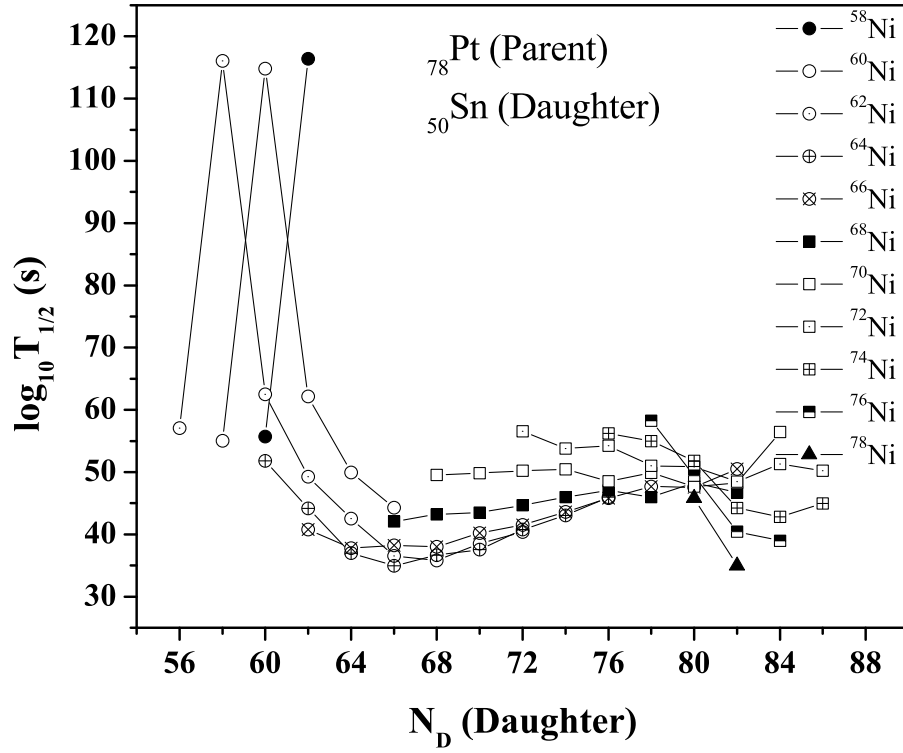
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Figure 3: Same as for Fig.1, but for $T_{1/2}$(s) alone, and for different Ni clusters emitted from various Pt parents.
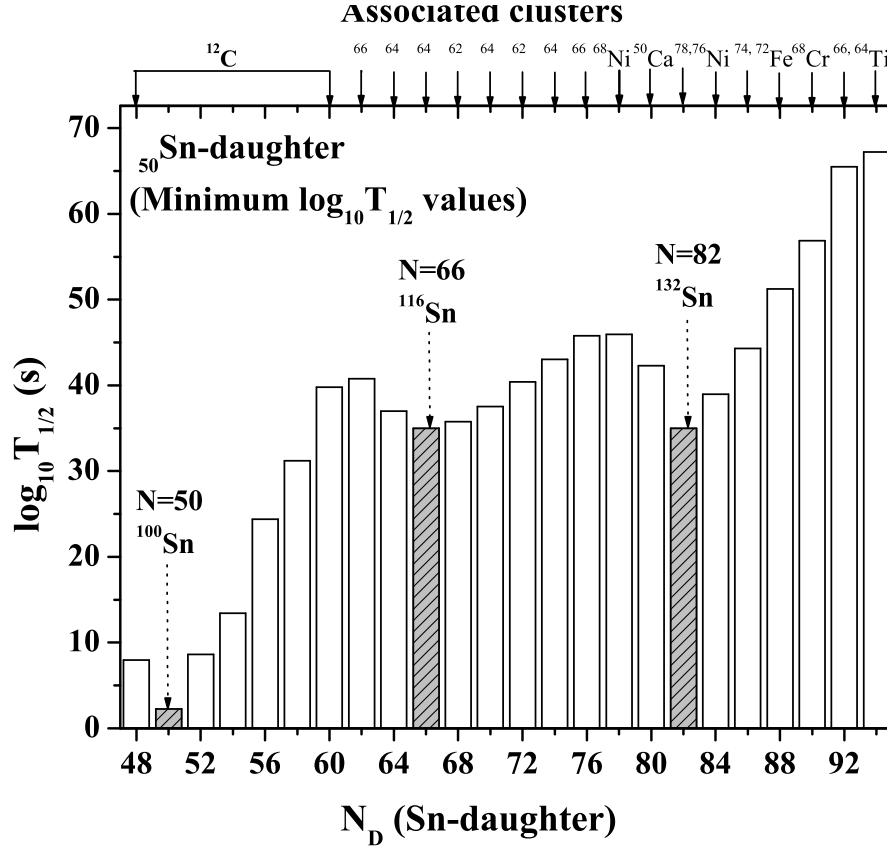
Figure 4: Histogram of $log_{10}T_{1/2}(s)$ versus $_{50}$Sn-daughter neutron number N_D for the most probable clusters emitted from various Ba to Pt parents with $_{50}$Sn as a daughter nucleus always, calculated on the basis of the PCM. The associated clusters are also shown on the top panel.

$N_D = 82$ is also evident with a minimum in the histogram at ^{132}Sn-daughter due to the emission of ^{78}Ni cluster from ^{210}Pt parent. The predicted half-life $log_{10}T_{1/2}(s) = 34.974$ (with a Q-value=119.292 MeV), which is beyond the limit of present day experiments. Thus, as expected, the strongest shell effects occur at $N_D = 50$ and 82. In addition, another minimum due to ^{64}Ni cluster emitted from ^{180}Pt parents could also be of interest for a closed shell (either spherical and/or deformed) at $N_D = 66$. This minimum is comparable to $N_D = 82$ case, with a predicted decay half-life also of nearly the same value ($log_{10}T_{1/2}(s) = 34.975$, with a Q-value=124.192 MeV), which by all means is again very large for experiments. Note from Fig. 2 that the decay half-life for ^{22}Mg emitted from ^{122}Sm ($log_{10}T_{1/2}(s) = 9.735$) lies in between the values for ^{12}C decay of ^{112}Ba and ^{78}Ni cluster from ^{210}Pt parent (or ^{64}Ni cluster emitted from ^{180}Pt parent), rather closer to the ^{12}C decay of ^{112}Ba.

4   Summary

The preformed cluster model (PCM) is used for the cluster decay calculations with $_{50}$Sn as a daughter nucleus always from various parents of Ba to Pt region. Thus ^{100}Sn and ^{132}Sn-daughter radioactivities is look for the most probable clusters (minimum decay half-life time) emitted from the rare-earth parents, and the presence of any new neutron magicity. The most probable clusters, respectively, with ^{100}Sn and ^{132}Sn daughters, are predicted to be ^{12}C from ^{112}Ba and ^{78}Ni from ^{210}Pt parent. Further possibilities with ^{100}Sn and ^{132}Sn daughters are also noticeable in ^{22}Mg and ^{50}Ca clusters emitted from ^{122}Sm and ^{182}Yb parents, respectively, as the second best new cases. In addition, a new shell is indicated at N_D=66 with ^{116}Sn-daughter due to ^{64}Ni cluster emitted from ^{180}Pt parents. However, at present these calculations seem to be more of an academic interest since the predicted half-life times, for at least the ^{116}Sn and ^{132}Sn-daughter radioactivities, are too large for experiments.

References

[1] H. J. Rose and G.A. Jones, Nature (London) **307**, 245 (1984).

[2] R. K. Gupta and W. Greiner, Int. J. Mod. Phys. E **3**, 335 (1994, Suppl.).

[3] R. Bonetti and A. Guglielmetti, in *Heavy Elements and Related New Phenomena*, edited by W. Greiner and R. K. Gupta, (World Scientific, Singapore, 1999), Vol. II, p. 643.

[4] R. Bonetti and A. Guglielmetti, Romanian Reports in Physics **59**, 301 (2007).

[5] A. Guglielmetti et al., Journal of Phys.: Conference Series **111**, 012050 (2008).

[6] A. Săndulescu, D. N. Poenaru and W. Greiner, Sov. J. Part. Nucl. **11**, 528 (1980).

[7] J. Maruhn and W. Greiner, Phys. Rev. Lett. **32**, 548 (1974).

[8] R. K. Gupta, W. Scheid, and W. Greiner, Phys. Rev. Lett. **35**, 353 (1975).

[9] R. Bonetti, A. Guglielmetti, V. L. Mikheev, and S. P. Tretyakova, Private communication; and to be published.

[10] D. N. Poenaru, R. Gherghescu and W. Greiner, Arxiv:1106.3271v1 (2011).

[11] D. C. Hoffman, T. M. Hamilton, M. R. Lane, Ch. 10 in Nuclear Decay Modes, pp. 393-432 (IOP Publishing, Bristol, 1996).

[12] D. N. Poenaru, W. Greiner and R. Gherghescu, Phys. Rev. C **47**, 2030 (1993);
D. N. Poenaru, W. Greiner, and E. Hourani, Phys. Rev. C **51**, 594 (1995).

[13] S. Kumar and R. K. Gupta, Phys. Rev. C **49**, 1922 (1994); ibid, Phys. Rev. C **51**, 1762 (1995). ibid J. Phys. G: Nucl. Part. Phys. **22**, 215 (1996).

[14] J. Walker et al., Phys. Rev. C **84**, 014319 (2011); and reference there in.

[15] S. Sarkar and M. Saha Sarkar, Phys. Rev. C **81**, 064328 (2010).

[16] Yu. Ts. Oganessian, *et al.*, Z. Phys. A **349**, 341 (1994).

[17] A. Guglielmetti, *et al.*, Phys. Rev. C **52**, 740 (1995).

[18] A. Guglielmetti *et al.* , Phys. Rev. C **56**, R2912 (1997).

[19] C. Mazzocchi, *et al.*, Phys. Lett. B **532**, 29 (2002).

[20] J. Gomez del Campo *et al.*, Phys. Rev. Lett. **61**, 290 (1988).

J. Gomez del Campo *et al.*, Phys. Rev. C **43**, 2689 (1991).

ibid C **57**, R457 (1998).

[21] Y. K. Vermani et al., Nucl. Phys. A**847**, 283 (2011); ibid J. Phys. G: Nucl. Part. Phys. **36**,105103 (2010), ibid G37 (2010) 015105; ibid,Europhys. Lett. 85, 62001 (2009);

R. K. Puri et al., Phys. Rev. C 54, R28 (1996); ibid J Comp Phys 162 (2000)245; ibid Phys Rev C 57 (1998)2744; ibid C58 (1998)320.

J. Singh et al., Phys. Rev. C 62, 044617 (2000).

[22] G. Audi, A. H.Wapstra and C. Thibault, Nucl. Phys. **A 729**, 337 (2003).

[23] P. Möller, et al., At. Data Nucl. Data Tables, **59**, 185 (1995).

[24] S. Kumar *et al.* Symp. on Nucl. Phys., Mumbai (India), Dec. 8-12, Vol. **46B**,

[25] S. Kumar *et al.*, J. Phys. G: Nucl. Part. Phys. **36**, 015110 (2009).

[26] S. S. Malik and R. K. Gupta, Phys. Rev. **C39**, 1992 (1989).

[27] S. Kumar and R. K. Gupta, Phys. Rev. **C55**, 218 (1997).

[28] I. Dutt and R. K. Puri, Phys. Rev. **C81**, 064609 (2010);and reference there in.

[29] J. Blocki, J. Randrup, W.J. Swiatecki, and C.F. Tsang, Ann. Phys. (NY) **105**, 427 (1977).

[30] R. K. Puri et al., Phys. Rev. C 45(1992) 1837 ; ibid. C **43** (1991) 315;ibid. C**47** (1993)561; ibid. Eur. Phys. J.**23** (2005) 429 ;ibid. J. Phys. G. **18** (1992) 903; ibid. G. **18** (1992)1533 ; ibid, Eur. Phys. J. A **3**, (1998) 277; ibid. Eur. Phys. J. A **8** (2000) 103; J. Phys. G. **18** (1992)1533; ibid Int. Mod. Phys E 1 (1992)269; ibid Phys Rev C 51 (1995)1568; ibid J Phys G 17(1991)1933; S. S. Malik et. al., Pramana J. **32** (1989)419; R. K. Puri, et al., ibid. A **575** (1994)733; I. Dutt et. al., Phys. Rev. C **81** (2010) 044615 ; C **81** (2010)064608; C **81** (2010)047601.

[31] S. Goyal et al., Phys. Rev C**83** (2011) 047601.

[32] A.D. Soode et al Phys Rev C70 (2004)034611; ibid C79 (2009)064618; ibid C73 (2006)067602; ibid J Phys G 37 (2010) 085102.

[33] Y. Vermani et al Phys Rev C 79 (2009)064613; S. Kumar et al , Phys Rev C 78 (2008) 064602; ibid C 81 (2010) 014601; ibid C 81 (2010) 014611; R. Chugh et al, Phys Rev C 82 (2010) 014603; A.D. Sood et al , Eur Phys J A 30(2006) 571.

[34] H. Kröger and W. Scheid, J. Phys. G **6**, L85 (1980).